NOTATION

x, y, Cartesian coordinates; η, transformed coordinate; u, v, velocity components; h, enthalpy; ρ, density; α, concentration; Z, relative velocity; τ, ε, relative enthalpy; ϑ, relative density; φ_1, φ_2, boundaries of jet; l_u, l_h, l_ρ, l_α, mixing lengths; a, mixing-length constant; c_p, specific heat at constant pressure; R, gas constant; c_1, integration constant. The subscript H refers to the outer and 0 to the inner boundary; < >, average value.

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A FLAT JET IN A GRANULAR BED OF FINITE HEIGHT

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UDC 532.546.6

The distribution of gas flows in the vicinity of the jet is discussed and the conditions of disruption of the static equilibrium of the bed, the formation and growth of a cavity, and the jet breakthrough of the bed are investigated qualitatively.

In the blowing of gas into a granular bed through an opening in its base one can distinguish three main modes of spread of the gas jet differing in a qualitative respect [1]. At low flow rates ordinary gas filtration occurs without disruption of the continuity of the bed. With an increase in the flow rate above some critical value, depending on the physical parameters of the particles and the gas, the size of the opening, and the geometry of the bed, the initial static equilibrium of the bed is disrupted: Near the orifice of the jet a cavity forms with a relatively small number of particles circulating in it, surrounded by granular material which is motionless as before, whose size grows with a further increase in the flow rate. When the flow rate exceeds some new critical value the equilibrium of the bed with the cavity also becomes impossible: The jet "breaks through" the bed with the establishment of a steady regime of the type studied in [2].

The pattern described also reflects the initial stages of development of a fountain in spouting beds, discussed in [3, 4], for example. The formation and growth of the cavity are analogous to the appearance and development of the "initial channel" in the mode of "internal" spouting [5, 6], as well as to the partially fluidized lower part of a granular bed in trough and conical apparatus [6-8]. The jet breakthrough of a bed is equivalent to this initial channel breaking through the entire bed and to the onset of a true spouting mode. In this case the critical values of the flow rate mentioned above correspond to the first two critical spouting velocities introduced in [9].

The theoretical analysis of the spread of a jet in a granular bed with disruption of the continuity of the latter and the analysis of the conditions of interchange of the above-indicated modes require the joint solution of two very complicated problems with an unknown boundary. First of all, we need to find the distribution of the hydraulic forces acting on the stationary granular material in the vicinity of the cavity on the part of the filtering gas in it, for which we must solve the filtration problem (nonlinear in the general case) on the distribution of gas flows in the bed. In addition, we must study the static equilibrium for a given distribution of hydraulic forces, and the unknown shape of the cavity must, in principle, be determined in the course of such an investigation. A rigorous solution of these interconnected problems is scarcely possible at present. Therefore, they are analyzed below with a number of simplifying assumptions only for a plane bed unbounded in a horizontal

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direction. In this case methods developed earlier in [10] are used to solve the first proble while the results of [11] are used for the second. The unification of the different methods of [10, 11] is thereby achieved and the possible paths for the analysis of the complicated problems arising in the investigation of the conditions for the onset of spouting in apparatus of the most varied form become clearer.

Below we assume that the same basic assumptions as in [10] are satisfied, i.e., we assume that the porosity of the bed near the cavity can be considered as uniform in a first approximation, while the hydraulic resistance to the gas flow can be considered as linear with respect to the filtration velocity, i.e., we investigate only the linear problem of filtration theory. In addition, we neglect variations in the dynamic gas pressure along the cavity, which are considerably less than the variations in gas pressure within the granular material, while we model the cavity itself, as in [10], with the help of a vertical profile $x' = 0$, $0 \leqslant y' \leqslant h$ in the complex flow plane $z' = x' + iy'$. The latter allow us to be abstracted from superfluous detailing of the shape of the cavity in the given stage, characterizing it by a single parameter, the height h, whose connection with the gas flow rate and the height of the granular bed is given below.

Under the adopted assumptions we obtain the following problem for the gas pressure within the bed (cf. [10]):

$$\Delta p = 0,\ \frac{\partial p}{\partial x} = 0\ (x = 0,\ 1 < y \leqslant H),\ \frac{\partial p}{\partial y} = -\alpha h u^\circ\ (y = 0),$$
$$p = p_0 > 0\ (x = 0,\ 0 \leqslant y \leqslant 1),\ p = 0\ (y = H). \tag{1}$$

Here we use dimensionless coordinates, introduced with a scale h, p_0 is the constant pressure inside the cavity, whose connection with the parameters of the problem is also determined below, and u° is the velocity of gas filtration in the bed undisturbed by the jet; the pressure above the bed is taken as the zero pressure reading. We note that in the case when u° is greater than the minimum fluidization velocity the problem (1) describes the distribution of gas flows near a jet introduced into a fluidized bed of finite height when the velocity of particle motion of the dense phase of the bed is neglected in comparison with the gas velocity [10].

As in [10], we introduce the velocity potential

$$\varphi = -\frac{p - p^\circ}{\alpha h},\ p^\circ = \alpha h u^\circ (H - y),\ \mathbf{u} = \mathbf{u}^\circ + \mathbf{v},\ \mathbf{v} = \nabla\varphi, \tag{2}$$

for which we obtain from (1) the problem

$$\Delta\varphi = 0,\ \frac{\partial\varphi}{\partial x} = 0\ (x = 0,\ 1 < y \leqslant H),\ \frac{\partial\varphi}{\partial y} = 0\ (y = 0),$$
$$\varphi = -\varphi_0 + u^\circ (H - y)\ (x = 0,\ 0 \leqslant y \leqslant 1),\ \varphi = 0\ (y = H),\ \varphi_0 = p_0/\alpha h. \tag{3}$$

In view of the symmetry of the problem (3) it is clear that it is sufficient to consider only the situation in the half-sheet $x \geqslant 0$, $0 \leqslant y \leqslant H$. This half-sheet of the plane $z = x + iy$ is mapped conformally onto the upper half-plane of the plane $\zeta = \xi + i\eta$ by the analytical function

$$\zeta = \operatorname{ch}\left(\frac{\pi z}{H}\right), \tag{4}$$

with the points iH, i, and 0 of the z plane changing into the points -1, $\beta = \cos(\pi/H)$, and 1 of the ζ plane, respectively.

Introducing the complex potential $\Phi = \varphi + i\psi$ and its derivative $F(\zeta) = d\Phi/d\zeta$, we obtain, by analogy with [10], the following problem for the analytical function $F(\zeta)$:

$$\operatorname{Re} F(\zeta) = \begin{cases} 0, & -\infty < \xi \leqslant -1, \\ f(\xi), & \beta \leqslant \xi \leqslant 1,\ \eta = 0,\ \beta = \cos\dfrac{\pi}{H}, \end{cases} \tag{5}$$

$$\operatorname{Im} F(\zeta) = 0, \ -1 < \xi < \beta, \ 1 < \xi < \infty, \ \eta = 0,$$

where

$$f(\xi) = \frac{\partial \varphi}{\partial \xi} = \frac{\partial \varphi}{\partial y}\frac{\partial y}{\partial \xi} = \frac{u^\circ H}{\pi\sqrt{1-\xi^2}}, \ \beta \leqslant \xi \leqslant 1, \ \eta = 0. \tag{6}$$

The function $F(\zeta)$ must be finite everywhere except, possibly, for the points $\xi = \beta$ and $\xi = 1$ of the real axis of the ζ plane, where $\Phi(\zeta)$ is finite.

The solution of the Hilbert problem (5) can be represented with the help of the Keldysh–Sedov equation [12] for any value of u°. Here we will investigate in detail only the jet flow in a stationary unventilated bed, when $u^\circ = 0$ and $\mathbf{u} = \mathbf{v}$. Replacing the limit $-\infty$ of variation of ξ in the first equation of (5) by $-a$, where a is some large positive number, and using the standard procedure [12], we obtain

$$F(\zeta) = \lim_{a\to\infty}\left\{\frac{(C + C'\zeta)\sqrt{a}}{\sqrt{(\zeta+a)(\zeta-\beta)(\zeta^2-1)}} + F(\infty)\sqrt{\frac{(\zeta+a)(\zeta-\beta)}{\zeta^2-1}}\right\} = C\,[(\zeta-\beta)(\zeta^2-1)]^{-1/2}. \tag{7}$$

The term containing $F(\infty)$ vanishes, since in the transition to the limit in (7) the quantity $F(\zeta)$ must remain finite, which is possible only when $F(\infty) = 0$, while the constant C' must be taken as equal to zero because the function $\Phi(\zeta)$, which represents the integral of $F(\zeta)$ by definition, must be finite at large ζ. For the complex velocity $U = u_x - iu_y$ we have from (4) and (7)

$$U(z) = \frac{d\Phi}{dz} = \frac{d\Phi}{d\zeta}\frac{d\zeta}{dz} = \frac{\pi C}{H}\left(\operatorname{ch}\frac{\pi z}{H} - \cos\frac{\pi}{H}\right)^{-1/2}. \tag{8}$$

The expressions for the components of the gas-filtration velocity are easy to obtain by introducing the real and imaginary parts into (8); they are not presented here due to their awkwardness.

The constant C can be determined in two ways. Integrating $u_y = -\operatorname{Im} U(z)$ over dy along the segment $x = 0$, $1 < y \leqslant H$ and using (2) and (3), we obtain

$$\varphi_0 = \frac{\pi C}{H}\int_1^H\left(\cos\frac{\pi}{H} - \cos\frac{\pi y}{H}\right)^{-1/2} dy = \sqrt{2}\,C\mathrm{K}\left(\cos\frac{\pi}{2H}\right), \tag{9}$$

where K(t) is a complete elliptic integral of the first kind. Then, integrating $u_x = \operatorname{Re} U(z)$ over $dy' = h\,dy$ along the segment $x = 0$, $0 \leqslant y \leqslant 1$ and equating the result to half the flow rate Q, by analogy with [10] we have

$$\frac{Q}{2} = \frac{\pi Ch}{H}\int_0^1\left(\cos\frac{\pi y}{H} - \cos\frac{\pi}{H}\right)^{-1/2} dy = \sqrt{2}\,Ch\mathrm{K}\left(\sin\frac{\pi}{2H}\right). \tag{10}$$

From (9) and (10) we obtain alternative representations for C:

$$C = \frac{\varphi_0}{\sqrt{2}}\mathrm{K}^{-1}\left(\cos\frac{\pi}{2H}\right), \quad C = \frac{Q}{2\sqrt{2}\,h}\mathrm{K}^{-1}\left(\sin\frac{\pi}{2H}\right). \tag{11}$$

Comparing these representations and using the definitions of p_0 and φ_0 in (2) and (3), we obtain an equation connecting the flow rate of the jet and the pressure inside the cavity:

$$p_0 = \frac{\alpha Q}{2}\mathrm{K}\left(\cos\frac{\pi}{2H}\right)\mathrm{K}^{-1}\left(\sin\frac{\pi}{2H}\right). \tag{12}$$

Performing the limiting transition $H \to \infty$, $z/H \to 0$ in (8) and (11), we arrive without difficulty at the equations for the jet flow in an unventilated granular bed of large height, obtained earlier in [10]. The situation when $H \to \infty$ but z/H is different from zero corresponds

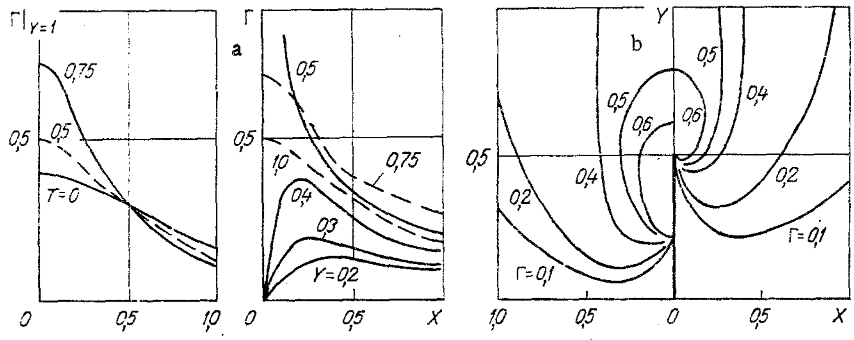

Fig. 1. Dependences of vertical component Γ of dimensionless filtration velocity at the exit from the bed (Y = 1) at different T and at different levels Y in the bed at T = 0.5 (a), and field of isotachs Γ = const at T = 0.25 and 0.5 (b)

to jet flow in a bed of finite height whose continuity is not disrupted.

To investigate the influence of the height of the cavity on the gas distribution in a bed of fixed height it is convenient to introduce the new coordinates

$$\begin{Bmatrix} X \\ Y \end{Bmatrix} = \frac{1}{Hh}\begin{Bmatrix} x' \\ y' \end{Bmatrix} = \frac{1}{H}\begin{Bmatrix} x \\ y \end{Bmatrix}. \tag{13}$$

In these coordinates the dimensionless height of the jet is obviously equal to $T = H^{-1}$, the upper surface of the bed corresponds to Y = 1, and Eqs. (8) and (11) have the previous form with the replacement of z/H by Z = X + iY. Profiles of the vertical component $\Gamma = u_y/u_0$ of the dimensionless velocity at different levels in the bed and isotachs Γ = const corresponding to two values of T are presented in Fig. 1; we introduce the velocity scale

$$u_0 = \pi Q/2\sqrt{2}\,Hh. \tag{14}$$

The curves of Fig. 1 reflect the "focusing" of the gas stream as the height of the cavity increases: The main mass of gas tends to exit into the space above the bed along the path of least resistance ($X\approx 0$) the more strongly, the greater the relative height T of the cavity. If the latter is comparable with unity, then the flow pattern differs very considerably from the pattern of "radial" outflow from a jet source in some angle $\theta\leqslant\pi$, discussed in [8, 11], for example.

The predominant flow of gas through the region of the bed lying directly above the cavity leads to a relative increase in the hydraulic force acting on the particles in this region, and thereby facilitates the disruption of the continuity of the bed, accompanied by the gradual growth of the cavity, and the jet breakthrough of the bed.

For a semiquantitative description of these effects (with the aim of the maximum simplification of the computations) we replace the true complicated distribution of gas velocity by a simpler one which correctly reflects the character of the variation of the flow with an increase in the cavity in a qualitative respect. To wit, let us consider a simple flow field with a vertical component of filtration velocity assigned by the equation

$$u_y^* \approx \frac{Q}{\theta h H Y}\left[1 - k^2\left(\frac{X}{Y}\right)^2\right], \quad X \ll Y, \tag{15}$$

where θ is a parameter playing the role of the effective spreading angle of the jet. For X = 0, Eq. (15) describes the flow occurring under conditions of radial spreading.

We determine the parameters θ and k from a comparison of Eq. (15) with the equation for the vertical component of the filtration velocity at the upper boundary of the bed near the point located above the entrance opening. From equations presented above we have

$$u_y\Big|_{Y=1,\,X\ll 1} \approx \frac{\pi Q}{2\sqrt{2}\,hH}\,K^{-1}\left(\sin\frac{\pi T}{2}\right)(1+\cos\pi T)^{-1/2}\left(1 - \frac{1}{4}\,\frac{\pi^2 X^2}{1+\cos\pi T}\right]. \tag{16}$$

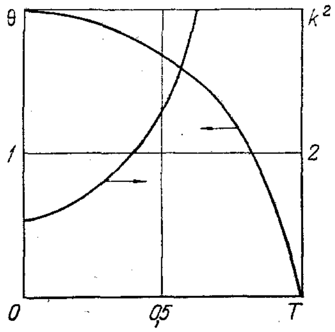

Fig. 2. Dependences of the effective spreading angle θ and the parameter k^2 on the dimensionless height T of the cavity.

Comparing (15) at Y = 1 with (16), for θ and k^2 we obtain the equations

$$\theta = \frac{2\sqrt{2}}{\pi} \mathrm{K}\left(\sin\frac{\pi T}{2}\right) (1 + \cos \pi T)^{1/2}, \tag{17}$$

$$k^2 = \frac{\pi^2}{4(1 + \cos \pi T)}, \tag{18}$$

which finally determine the "quasiradial" flow (15).

We note that the angle θ thus determined differs from π even for flow in a bed whose continuity is not disrupted at all (there is no cavity). In fact, as $T \to 0$ we obtain $\theta \to 2 < \pi$ from (17). The indicates, as one would expect, that the symmetrical radial spreading into the total angle accessible to the gas, even if it occurs in the depths of the bed, is considerably distorted near its upper boundary. The dependences of θ and k^2 on T are presented in Fig. 2.

Now let us investigate the static equilibrium of a granular bed containing a cavity and find the critical values of the flow rate Q_1 and Q_2, upon reaching which the initial plastic shear of the bed occurs (a cavity first forms) and its jet breakthrough is accomplished (static equilibrium ceases to be possible at all), respectively. For this purpose we use the method of [11], approximating the distribution of the vertical component of the filtration velocity by Eq. (15). Within the framework of the approximate model in [11] we consider as the region of plastic shear a region, symmetric relative to the plane X = 0, with flat boundaries inclined to the vertical at an angle $\psi^\circ = \arctan (1/\nu_m)$, where $\nu_m = \nu_m(T)$ is the value of the parameter ν achieving a minimum of the function†

$$G(T, \nu) = \frac{\nu^2}{\nu^2 - k^2/3} \frac{1 + b + a\nu}{2 + b + a\nu} \frac{1 - T^{2+b+a\nu}}{1 - T^{1+b+a\nu}}. \tag{19}$$

Here k^2 is defined in (18) while a and b are coefficients which depend on φ_f — the angle of internal friction of the free-flowing material, calculated in [11]. In this case in the notation adopted here the equation connecting the value of u, the quantity u_y^* or u_y at X = 0 and Y = 1, with the characteristic dimensionless height T of the cavity is written in the form

$$u = u_* G(T, \nu_m), \quad u_* = (1 - \varepsilon)\gamma/\alpha, \tag{20}$$

where u_* is the minimum fluidization velocity. The corresponding values of the flow rate Q for u can be determined from (20) using an equation which follows from (16), for example. As a result, we have an expression for the dimensionless flow rate q at which a granular bed containing a cavity with a height T is in a state of static equilibrium:

†We note the difference between the designations in this report and in [11]. The dimensionless height z of the cavity, consisting of the ratio of its dimensional height to the height of the bed above the cavity, was introduced in [11]. Therefore, $z = T(1 - T)^{-1}$. The substitution of this expression into the corresponding equation in [11] leads to (19).

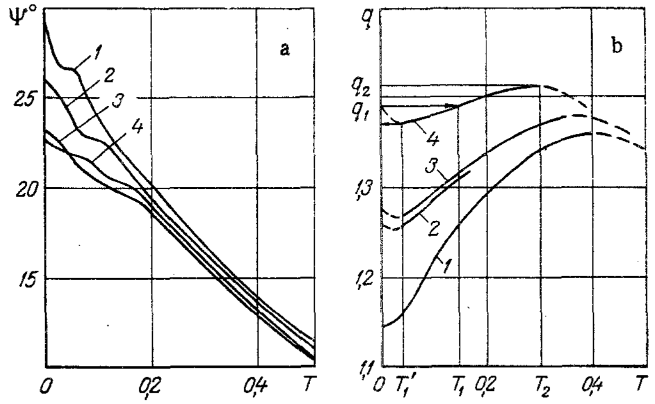

Fig. 3. Dependence of angle $\psi°$ of inclination of boundaries of the region of plastic flow (a) and of dimensionless flow rate q (b) on the parameter T. Curves 1-3) model with $b \neq 0$ for different φ_f equal to 30, 45, and 60°, respectively; curve 4) model with $b = 0$ for $\varphi_f = 30°$. Arrows mark a discontinuous change in the state of the bed and dashes correspond to unrealized states.

$$q = \frac{Q}{u_* hH} = \frac{2\sqrt{2}}{\pi} \mathrm{K}\left(\sin\frac{\pi T}{2}\right)(1+\cos\pi T)^{1/2} G(T, \nu_m). \tag{21}$$

The pressure drop in the bed is obviously determined by Eq. (12). Using (21) and introducing $H = T^{-1}$, for the dimensionless drop Δp we have

$$\Delta p = \frac{p_0}{\alpha u_* hH} = \frac{1}{2}\mathrm{K}\left(\cos\frac{\pi T}{2}\right)\mathrm{K}^{-1}\left(\sin\frac{\pi T}{2}\right) q(T). \tag{22}$$

Concrete calculations of the quantities $\psi°$, q, and Δp were made in accordance with two models proposed in [11]. In the first of them one takes $b = 0$, while in the second one uses the dependence of b on φ_f calculated in [11]. In the case when the size of the angle $\psi°$ is comparable with φ_f, the first model is evidently preferable (see [11]); when $\psi° \ll \varphi_f$, however, one can expect that the second model will give better results. The dependences of $\psi°$ on φ_f at three values of T and $b \neq 0$ are presented in Fig. 3a and one of the curves corresponding to the model with $b = 0$ is also shown there. It is seen that the region of plastic shear, which converts the bed into a new state of static equilibrium with a higher cavity upon a small increase in q, narrows considerably as the cavity grows.

The corresponding dependences of the quantity q on T are presented in Fig. 3b. The first critical value q_1 of this quantity, at which the continuity of the bed is first disrupted, is determined by the points of intersection of these dependences with the ordinate axis. In this case, when the angle of internal friction is large enough, a cavity with a finite height T_1 forms at once in the bed: The discontinuous transition from a state in which the continuity of the bed is not disrupted to a state of equilibrium with such a cavity is shown by an arrow in Fig. 3b. With a further smooth increase in the flow rate the height of the "equilibrium" cavity increases monotonically until the "maximum" value T_2 is reached, which gives a maximum q_2 of the corresponding curve q(T) in Fig. 3b. When the flow rate (21) exceeds the value q_2 the state of static equilibrium proves to be impossible at all, and the instantaneous jet breakthrough of the bed occurs; states corresponding to the section of the q(T) curve to the right of the line $T = T_2$ are not realized in practice.

If the flow rate is smoothly decreased, starting with some state of static equilibrium with a cavity, then the height of the cavity will decrease monotonically until the "minimum" value $T_1' < T_1$ giving a minimum of the curve q(T) is reached. After this the bed abruptly recovers its continuity — the cavity disappears (also marked by an arrow on one of the curves of Fig. 3b). Thus, the variation in the state of the bed in the region of $0 \leqslant T \leqslant T_1$ is characterized by a kind of hysteresis, resembling the hysteresis in processes of dry friction,

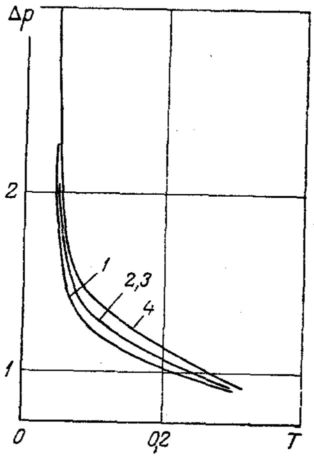

Fig. 4. Dependences of dimensionless pressure drop Δp on T with $T_0 = 0.05$; curves 1-4) see Fig. 3. The curves break off at $T = T_2$.

e.g., when the static coefficient of friction is higher than the dynamic one.

In the region of $q < q_1$ the pressure drop in the bed is proportional to q, with the proportionality factor going to infinity as $T \to 0$, as is easily seen from (12) or (22). Actually, the quantity T is never equal to zero: In a granular bed without a cavity it takes a finite value T_0 characterizing the size of the opening through which the jet escapes. When the initial packing of the bed particles is free-flowing, T_0 is also determined by processes of overpacking of particles near the orifice of the jet with an increase in the flow rate. In the region of $q_1 < q <_2$ the quantity Δp varies in accordance with (22), in which q is considered as a function of T defined implicitly by Eq. (21) or by the curves of Fig. 3b. As is easy to see, at $q = q_2$ the value of Δp characterizes the pressure drop at the moment of breakthrough of the bed. Dependences of Δp on T, obtained from (22) with $T_0 =$ 0.05 and corresponding to the curve of q(T) in Fig. 3b, are presented in Fig. 4.

Thus, the analysis conducted provides a physical understanding of the complicated processes accompanying the change in the state of the bed and its disruption under the action of the filtering flow, and the conclusions following from it and involving both the character of this change with a change in the flow rate and the dependence of the pressure drop on the flow rate are confirmed, in a qualitative respect, by the entire complex of experimental data accumulated in the investigation of jet flows in granular beds [1], the mechanism of fluidization in conical apparatus [7, 8], and the development of a fountain in different types of spouting beds [3-6, 9].

Further supplementary work is needed, however, for a more exact quantitative description of these processes under different conditions and the construction of the corresponding engineering methods of calculation. The analysis presented is inadequate in this respect, of course. Moreover, the point here is not only that approximate equations are used for the filtration velocity, the hydraulic resistance is assumed to be linear with respect to this velocity and the porosity to be uniform, the boundaries of the region of plastic shear are taken a priori as plane (or as conical in the axisymmetric case [11]), and the jet is modeled by a profile in the plane of flow for which the connection of the height with the true geometry of the cavity is unclear, generally speaking, but also that the bulging of the material at the free surface of the bed above the cavity, which affects the gas distribution, processes of overpacking and consolidation of the particles, the effective value of the angle of internal friction, etc., are not taken into account.

Further progress primarily requires the setting up of more-delicate tests specially directed both at the quantitative testing of the conclusions following from the model in [11] and the present report, particularly at the choice between the two variants of this model, and at the refinement of the model itself and the creation on its basis of simplified calculating schemes, which would possibly contain some empirical constants admitting of a simple experimental determination.

NOTATION

a, b, functions calculated in [11]; C, C', constants in (7); F, derivative of the complex potential; f, function in (6); G, function defined in (19); H, dimensionless height of bed; h, height of cavity; k, coefficient introduced in (15); p, p_0, pressure inside bed and in cavity; Δp, dimensionless pressure drop; Q, q, dimensional and dimensionless jet flow rates; q_1, q_2, critical values; T, dimensionless height of cavity; T_0, T_1, T_1', T_2, characteristic values of T; **u**, **v**, filtration velocities; u°, u_*, initial filtration velocity in the bed and minimum fluidization velocity; u_0, velocity scale introduced in (14); u^*, velocity scale introduced in (14); u^*, velocity of fictitious flow defined in (15); U, complex velocity; Z = X + iY, z = x + iy, dimensionless coordinates; z' = x' + iy', dimensional coordinates; α, coefficient of hydraulic resistance; β, parameter from (5); $\Gamma = u_y/u_0$; γ, specific weight of particles' material; ε, porosity; $\zeta = \xi + i\eta$, coordinates in the plane obtained from z = x + iy as a result a of conformal transformation; $\nu = \tan^{-1}\psi^\circ$; ν_m, value of ν giving a minimum of the function G; Φ, φ_f complex and real flow potentials; φ, angle of internal friction; ψ, stream function; ψ°, angle of inclination of boundaries of the region of plastic flow to the vertical.

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ANALYSIS OF THE PARAMETERS OF DISCRETE PARTICLE MOTION IN AXISYMMETRIC TURBULENT IMPINGING JETS

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The parameters of discrete particle motion in axisymmetric turbulent impinging air jets are determined.

The method of impinging jets is used extensively at the present time [1] to intensify the heat and mass transfer in technological processes and apparatus. Impinging air jets are quite efficient even for the preparation of concrete mixtures [2]. The self-similar section of sand and cement particle acceleration with a quadratic drag law starting from the plane of the impinging jet is governing in the jet agitation of a concrete mixture. Because of the elastic strain of the air flux at sites of shock merger of the jet, and in the presence of inertial forces, the sand and cement particles from one jet penetrate into an other and are decelerated. These particles do not succeed in being accelerated in the opposite direction since coaxiality of the nozzle sources is spoiled in the subsequent times because of the structural peculiarities of the continuous operation mixer for the preparation of concrete mixtures [2].

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